**Semantics of predicate logic**

In semantics, we expect something like truth tables. By ‘equivalent,’ we mean that we should be able to prove soundness and completeness, as we did for propositional logic – although a fully fledged proof of soundness and completeness for predicate logic is beyond the scope of this book. Before we begin describing the semantics of predicate logic, let us look more closely at the real difference between a semantic and a proof-theoretic account. In proof theory, the basic object which is constructed is a proof. Let us write $\Gamma$ as a shorthand for lists of formulas $\varphi 1$, $\varphi 2$,...,$\varphi n$. Thus, to show that $\Gamma \quad \psi$ is valid, we need to provide a proof of $\psi$ from $\Gamma$. Yet, how can we show that $\psi$ is not a consequence of $\Gamma$? Intuitively, this is harder; how can you possibly show that there is no proof of something? You would have to consider every ‘candidate’ proof and show it is not one. Thus, proof theory gives a ‘positive’ characterisation of the logic; it provides convincing evidence for assertions like ‘$\Gamma \quad \psi$ is valid,’ but it is not very useful for establishing evidence for assertions of the form ‘$\Gamma$ $\varphi$ is not valid.

Semantics, on the other hand, works in the opposite way. To show that $\psi$ is not a consequence of $\Gamma$ is the ‘easy’ bit: find a model in which all $\varphi i$ are true, but $\psi$ isn’t. Showing that $\psi$ is a consequence of $\Gamma$, on the other hand, is harder in principle. For propositional logic, you need to show that every valuation (an assignment of truth values to all atoms involved) that makes all $\varphi i$ true also makes $\psi$ true. If there is a small number of valuations, this is not so bad. However, when we look at predicate logic, we will find that there are infinitely many valuations, called models from hereon, to consider. Thus, in semantics we have a ‘negative’ characterisation of the logic. We find establishing assertions of the form ‘$\Gamma \quad \psi$’ ($\psi$ is not a semantic entailment of all formulas in $\Gamma$) easier than establishing ‘$\Gamma \quad \psi$’ ($\psi$ is a semantic entailment of $\Gamma$), for in the former case we need only talk about one model, whereas in the latter we potentially have to talk about infinitely many. All this goes to show that it is important to study both proof theory and semantics. For example, if you are trying to show that $\psi$ is not a consequence of $\Gamma$ and you have a hard time doing that, you might want to change your strategy for a while by trying to prove the validity of $\Gamma \ \psi$. If you find a proof, you know for sure that $\psi$ is a consequence of $\Gamma$. If you can’t find a proof, then your attempts at proving it often provide insights which lead you to the construction of a counter example.